In the last several decades, the theory of spline functions and their applications have greatly influenced numerous fields of applied mathematics, most notably, computational mathematics, wavelet analysis, and geometric modeling. Many books and monographs have been published studying real variable spline functions with a focus on their algebraic, analytic, and computational properties. In contrast, this book is the first to present the theory of complex harmonic spline functions and their relation to wavelet analysis with applications to the solution of partial differential equations and boundary integral equations of the second kind. The material presented in this book is unique and interesting. It provides a detailed summary of the important research results of the author and his group, and others in the field as well.

The book is organized into four chapters. Chapter I provides a rigorous study of the functional and geometrical properties of complex harmonic spline functions. Specifically, it contains the general theory of the interpolating and quasi-interpolating complex spline functions on the boundary of the unit disk. It also contains a discussion how the boundary values of complex harmonic spline functions influence their interior behavior. An algorithm for the computation of complex harmonic spline functions is also provided. In Chapter II various types of periodic quasi-wavelets are constructed using real and complex spline functions as generators. The orthogonality and least number of terms in the decomposition formulas for periodic quasi-wavelets, which are very important in applications, are thoroughly discussed. In Chapter III, the author applies periodic quasi-wavelets to solve boundary value problems for the two-dimensional Helmholtz equation by reducing it to a Fredholm integral equation of the second kind with a weakly singular kernel. Under certain smoothness conditions on the coefficients and the stiffness matrix being given, it is proved that the order of complexity of this algorithm is $O(N)$, where N represents the number of unknowns. In Chapter IV another type of periodic wavelets is constructed. These explicitly given wavelets possess the following important properties: interpolation, localization, symmetry, regularity up to any prescribed order, real-valued, and biorthogonal. Some illustrative examples are provided.

In summary, this book is a rigorous presentation of the numerous interesting mathematical properties and physical applications of complex harmonic spline functions, which is suitable not only as a reference source but also as a textbook for a special topics course or seminar. We are delighted to see the publication of this book and hope that it will foster new research and applications of complex harmonic splines and wavelets. We enthusiastically recommend it to the mathematics and engineering communities.

Charles A. Micchelli
E-mail: cam@watson.ibm.com
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Boris Osilenker, *Fourier Series in Orthogonal Polynomials*, World Scientific, Singapore, 1999, vi + 287 pp.

A classical topic in approximation theory is polynomial approximation. The best polynomial approximation of a function in a Hilbert space is obtained by taking the orthogonal projection of the function onto the linear space of polynomials of degree at most n. If one uses a basis of orthogonal functions, then this orthogonal projection becomes a partial sum of the Fourier series. Hence orthogonal polynomials and Fourier series in orthogonal polynomials are the building blocks for best polynomial approximation in a Hilbert space. The underlying theory is quite classical by now and is covered in the standard books on orthogonal polynomials (Szegő's *Orthogonal Polynomials* from 1939 and Freud's *Orthogonal Polynomials* from 1971, to name two of the most relevant books).

The book under review basically covers the standard material and adds some new material, mostly from the author's research results. Chapter 1 deals with some preliminaries, such as topics from function theory of a real variable, some topics from functional analysis (Banach–Steinhaus theorem), interpolation theorems (Riesz–Thorin and Marcinkiewicz), and a short section on the Hardy–Littlewood maximal function. Chapter 2 is where orthogonal systems in a Hilbert space enter the picture. Here we find the typical results on Gram matrices, Gram–Schmidt orthogonalization, and completeness. Orthogonal polynomials and their extremal properties are of course treated in more detail. In particular we find Christoffel's theorem regarding orthogonal polynomials with weight $\rho(x)\,d\mu(x)$ in terms of orthogonal polynomials with measure μ, where ρ is a nonnegative polynomial. The three-term recursion relation and related Jacobi matrices are given, with a proof of Favard's theorem based on the spectral theorem for self-adjoint operators. The reproducing kernel for polynomial approximation is referred to as the Dirichlet kernel in this book (the Christoffel–Darboux kernel would have been more appropriate). The Jacobi polynomials are then treated in some detail (explicit formulas, the recurrence relation, the differential equation, uniform estimates, asymptotic properties and weighted estimates). Next, the author explains different approaches to obtaining estimates for general orthogonal polynomials on an interval $[a, b]$ and their Christoffel functions, including an approach using the recursion relation.

Chapter 3 deals with convergence and summability of Fourier series. Convergence in norm, pointwise convergence, and almost everywhere convergence are considered (but apparently not the uniform convergence), together with estimates of the Lebesgue function. The chapter ends with the convergence of the arithmetic (Cesàro) means of the partial sums of the Fourier series. Chapter 4 looks at the Fourier series in L^p $(1 < p < \infty)$ and in the space C of continuous functions. First it is shown that for each orthonormal system of polynomials, there is a continuous function such that its Fourier series is not uniformly convergent. Then some technical estimates of the Lebesgue function are proved and some results of the author on strong summability in L^p are presented. Fourier series in L^1 are the topic of Chapter 5, with Poisson–Abel summability, which culminates in the analog of Fatou's theorem on differentiated Fourier series. The final chapter contains work of the author on trilinear kernels, extending the Christoffel–Darboux kernel (or Dirichlet kernel) which is a bilinear kernel. This trilinear kernel is useful for finding (generalized) product formulas for orthogonal polynomials and for investigating a generalized translation operator in orthogonal polynomials.

Overall, this book treats Fourier series in orthogonal polynomials in a rather classical approach. Only orthogonal polynomials on a finite interval are considered. Laguerre and Hermite polynomials are not covered, and in general there is no treatment in this book of Fourier series in orthogonal polynomials on infinite intervals. The newer results of the author are usually quite technical. The English language is often very poor and one would expect the publisher to put more effort into helping out authors whose native language is not English.

Walter Van Assche
E-mail: Walter.VanAssche@wis.kuleuven.ac.be
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Proceedings

Applications and Computation of Orthogonal Polynomials, W. Gautschi, G. H. Golub, and G. Opfer, Eds., International Series of Numerical Mathematics **131**, Birkhäuser, Basel, 1999, xiii + 268 pp.

A workshop on *Applications and Computation of Orthogonal Polynomials* took place between March 22 and 28, 1998, at the Oberwolfach Mathematical Research Institute. This volume contains refereed versions of 18 papers presented at (or submitted to) the conference. The volume is dedicated to the memory of Günther Hämmerlin who initiated, directed, and co-directed a series of Oberwolfach conferences on numerical integration (a picture is included). Topics covered are Gauss quadrature, least-squares polynomial approximation, Szegő polynomials, Lanczos' method, Müntz orthogonal polynomials, and various papers dealing with classical and nonclassical orthogonal polynomials.

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